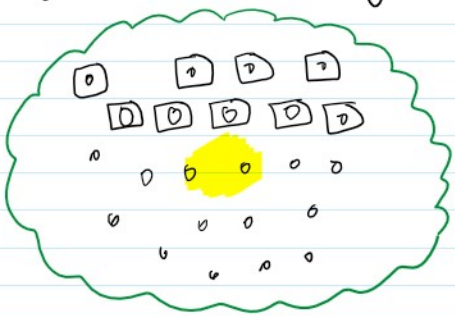
Ch 8

CH : Measures of Central Tendency

→ One of the most important objective of statistical analysis is to obtain a single representative value. the one single value that describes the characteristics of the complex & varied mass of data is called **average** or **central value**.

→ It is generally located in the centre or middle of the observations. Thus it is called **Measure of Central Tendency** or Measure of Location.

→ It is defined as an **average** to find one **single figure** that describes the characteristics of population.



Represent " 1 value "

* OBJECTIVES / FUNCTIONS of Averages *

→ To **represent** the salient features of mass complex data

→ To facilitate **comparison**

→ To **know about the universe** from a sample

→ To help in **decision making**

→ To trace **mathematical relationship**

* Characteristics of a Representative Average *

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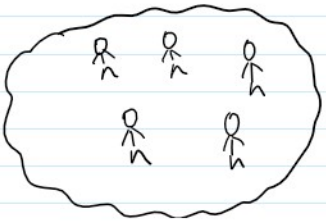


characteristics

class representative (Average)

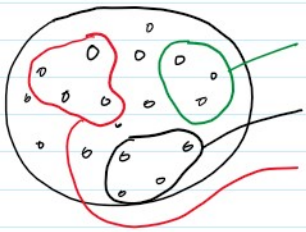
(Average)

(i) It should be simple to calculate and easy to understand

(ii) It should be rigidly defined

(iii) It should be based on all the observations

(iv) It should be least affected by fluctuations of sample



Sample 1 — Average 2

Sample 2 — Average 2.02

Sample 3 — Average 2.03

(v) It should not be affected much by extreme Values

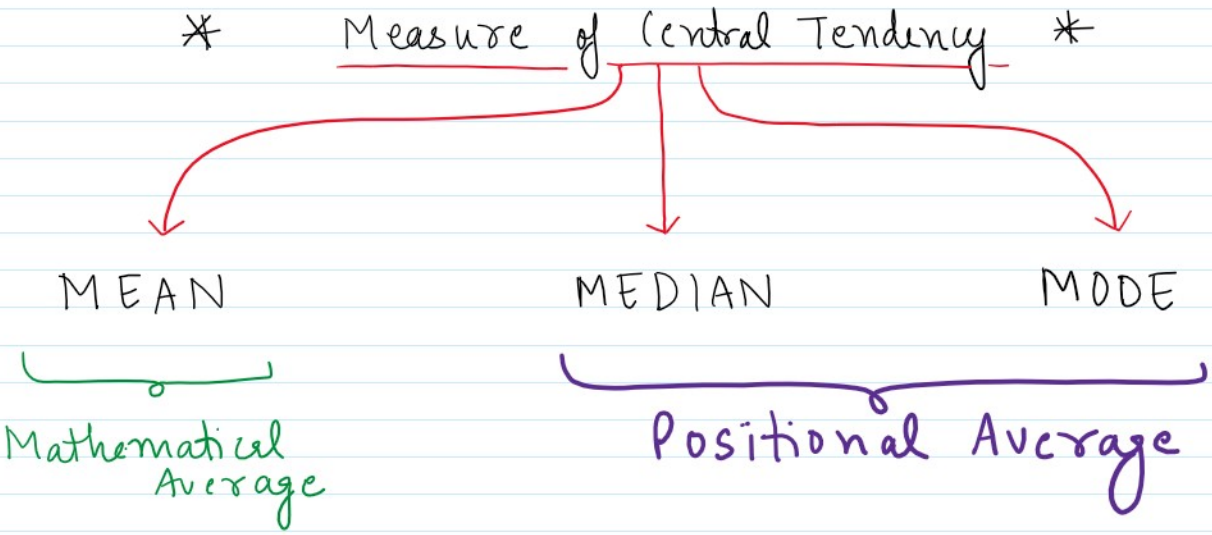
2, 1, 2, 1, 0, 3, 1, 4, 99

extreme Value

* Three Orders of Measurement *

- Measures of first order → First stage of statistical calculation (Measures of Central Tendency)
- Measures of second order → Second stage of statistical calculation (Measures of Dispersion)
- Measures of third order → Third stage of statistical calculation (Measures of Skewness & Kurtosis)

* Measure of Central Tendency *



I ARTHMETIC MEAN

→ Individual Series
→ Discrete Series
→ Continous Series

- Direct Method
- Short Cut Method (Assumed Mean Method)
- Step Deviation Method

* INDIVIDUAL SERIES *

eg ① Following are the marks of 5 students :

2, 4, 3, 8, 6

Find A.M

Method ① Direct Method

$$\bar{X} = \frac{\Sigma X}{N}$$

ΣX → Sum of all observations

N → Number of observations

$$= \frac{2+4+3+8+6}{5}$$

$$= \frac{23}{5} = 4.6 \text{ marks } (\smile)$$

Method ② Short Cut Method (Assumed Mean Method)

$$\underset{(A.M)}{\bar{X}} = A + \frac{\Sigma d}{N}$$

here
A = Assumed Mean
Σd = Sum of deviation from assumed mean
N = No. of observations

Let **Assumed Mean = 6**

X	d = X − A
2	−4
4	−2
3	−3
8	2
6	0
	$\Sigma d = (-)7$

$$\bar{X} = A + \frac{\Sigma d}{N}$$

$$= 6 \ (-) \ \frac{7}{5}$$

$$= 6 \ (-) \ 1.4$$

$$= 4.6 \text{ marks}$$

Method ③ Step Deviation Method

eg :- Following are the observations :-

1010 , 1030 and 1020

Find Mean.

Sol:- $$\bar{X} = A + \frac{\Sigma d'}{N} \times c$$

here A = Assumed Mean

$\Sigma d'$ = Sum of deviations

N = No. of Observations

c = common factor

X	$d = X - A$	$d' = d/c$
1010	(-) 20	(-) 2
(A) 1030	0	0
1020	(-) 10	(-) 1
		$\Sigma d' = (-)3$

Let $C = 10$

$$\bar{X} = A + \frac{\Sigma d'}{N} \times C$$

$$= 1030 - \frac{3}{3} \times 10$$

$$= 1030 - 10$$

$$= 1020$$

II. DISCRETE SERIES

eg :-

Marks	10	20	30	40	50
No. of students	5	10	40	20	25

Method 1 : DIRECT METHOD

$$\bar{X} = \frac{\Sigma fX}{\Sigma f}$$

Marks (X)	No. of Students (f)	fX
10	5	50
20	10	200
30	40	1200
40	20	800
50	25	1250
	$\Sigma f = 100$	$\Sigma fX = 3500$

$$\bar{X} = \frac{\Sigma fX}{\Sigma f} = \frac{3500}{100} = 35$$

$$\bar{X} = \frac{\Sigma f}{\Sigma f} = \frac{\quad}{100} = \quad$$

② Method II : Short Cut Method (Assumed Mean method)

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

Marks (X)	No. of students (f)	d = X − A	fd
10	5	(−) 20	(−) 100
20	10	(−) 10	(−) 100
(A) 30	40	0	0
40	20	10	200
50	25	20	500
	$\Sigma f = 100$		$\Sigma fd = 500$

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 30 + \frac{500}{100} = 30 + 5 = 35$$

Method ③ :- STEP DEVIATION METHOD

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times c$$

Marks (x)	No. of student (f)	d = X − A	d' = d/c	fd'
10	5	(−) 10	(−) 1	−5
A 20	10	0	0	0
30	40	10	1	40
40	20	20	2	40
50	25	30	3	75
	$\Sigma f = 100$			$\Sigma fd' = 150$

$c = 10$

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times c$$

$$= 20 + \frac{150}{100} \times 10$$

$$= 35$$

III. CONTINOUS SERIES

eg :-

Marks	0 - 4	4 - 8	8 - 12	12 - 16
No. of Students	4	8	2	1

find Mean

Method 1 : DIRECT METHOD

$$\bar{X} = \frac{\Sigma f m}{\Sigma f}$$

Marks	No. of Students (f)	Mid. points (m)	f m
0 - 4	4	2	8
4 - 8	8	6	48
8 - 12	2	10	20
12 - 16	1	14	14
	$\Sigma f = 15$		$\Sigma fm = 90$

$$\bar{X} = \frac{\Sigma f m}{\Sigma f} = \frac{90}{15} = 6$$

Method (2) Short Cut Method (Assumed Mean Method)

$$\bar{X} = A + \frac{\Sigma f d}{\Sigma f}$$

Marks	No. of Students (f)	Mid Value (m)	d = m - A	f d
0 - 4	4	2 (A)	0	0

0 - 4	4	2 (A)	0	0
4 - 8	8	6	4	32
8 - 12	2	10	8	16
12 - 16	1	14	12	12
	$\Sigma f = 15$			$\Sigma fd = 60$

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 2 + \frac{60}{15}$$

$$= 2 + 4 = 6$$

Method (3) :- STEP DEVIATION METHOD

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times c$$

Marks	No. of students (f)	Mid values (m)	$d = m - A$	$d' = d/c$	fd'
0 - 4	4	2	- 8	- 2	- 8
4 - 8	8	6	- 4	- 1	- 8
8 - 12	2	10 (A)	0	0	0
12 - 16	1	14	4	1	1
	$\Sigma f = 15$	$c = 4$			-15

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times c$$

$$= 10 + \frac{(-15)}{15} \times 4$$

$$= 10 - 4 = 6$$

$= 10 - 4 = 6$

* SPECIAL CASES *

(i) Open ended class.

Marks	No. of students.
Below 10	5
10 - 15	8
15 - 20	3
20 - 25	2
Above 25	6

Here,

Marks	No. of Students
5 - 10	5
10 - 15	8
15 - 20	3
20 - 25	2
25 - 30	6

Rest all steps will be same

(ii) Cumulative frequency distribution

Here, Convert it into "normal class Interval" distribution.

After that all steps will be same

(iii) When Mid Values are given

Mid Values	f
5	3
15	2
25	8
35	1

→ Solve DIRECTLY

(iv) Inclusive Series

Marks	No. of Students
20 - 29	3
30 - 39	8

20 - 29	3
30 - 39	8
40 - 49	4
50 - 59	2

Here,

NO NEED TO CONVERT

Directly take **Mid Values** and solve as per the steps.

(v) **Unequal class Intervals**

Marks	No. of students
0 - 10	5
10 - 20	3
20 - 40	8
40 - 70	2

Here,

NO NEED TO CONVERT it into equal class Interval

Directly calculate **Mid Values** and solve as per the steps.

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* Properties of Arithmetic Mean *

(1) **The sum of deviations** of the items from the A.M is always equal to **ZERO**

eg :- Following are the marks of 5 students

25, 5, 15, 20, 10

Prove the above property

Sol:-

X	$X - \bar{X}$
25	10
5	(-)10
15	0
20	5
10	(-)5

→ deviations from A.M

15	0
26	5
10	(-) 5
75	$\Sigma(x-\bar{x}) = 0$

→ Sum of deviations from A.M

$$\bar{X} = \frac{\Sigma X}{N} = \frac{75}{5} = 15$$

② The sum of the squares of the deviations from A.M is LEAST

eg :- Following are the marks of 5 students

3, 2, 1, 4, 5

Prove the property

Case A			Case B		
X	X − 3	$(X-3)^2$	X	X − 1	$(X-1)^2$
3	0	0	3	2	4
2	−1	1	2	1	1
1	−2	4	1	0	0
4	1	1	4	3	9
5	2	4	5	4	16
		10			30

$$\underset{(A.M)}{\bar{X}} = \frac{\Sigma X}{N} = \frac{15}{5} = 3$$

③ A.M is affected by change in Origin

When a value is added or deducted from all observations.

eg :- Following are the observations.

2, 7, 6, 1

Find A.M

Sol :- $\bar{X} = \frac{\Sigma X}{} = \frac{2+7+6+1}{} = 16$

Find A.M

Sol :- $\bar{X} = \frac{\Sigma X}{N} = \frac{2+7+6+1}{4} = \frac{16}{4}$

$= 4$

Now "Add 5" in all observations (+5)

New observations are :- 7, 12, 11, 6

New A.M $= \frac{7+12+11+6}{4} = \frac{36}{4} = 9$

(4) A.M is affected by change in Scale

When a value is multiplied or divided to all observations

Eg :- Following are the observations :-

6, 2, 8, 12

Find A.M

Here $\bar{X} = \frac{\Sigma X}{N} = \frac{6+2+8+12}{4} = \frac{28}{4}$

$= 7$

Divide all observations by 2 (÷2)

New observations = 3, 1, 4, 6

New $\bar{X} = \frac{\Sigma X}{N} = \frac{3+1+4+6}{4} = \frac{14}{4} = 3.5$

(5) If all observations are "K"

then A.M is K

eg :- observations : 7, 7, 7, 7, 7

Find A.M $\Rightarrow \frac{7+7+7+7+7}{5} = \frac{35}{5}$

$= 7$

(6) **Combined Mean**

$$\overline{X}_{1.2} = \frac{N_1 \cdot \overline{X}_1 \ (+) \ N_2 \cdot \overline{X}_2}{N_1 + N_2}$$

$\overline{X}_{1.2}$ — Combined Mean

N_1 = No. of observations in 1st group

N_2 = " " " " 2nd group

$\overline{X}_1$ = A.M of 1st group

$\overline{X}_2$ = A.M of 2nd group

eg :- Group A

A.M = 40 and N = 60

Group B

A.M = 35 and N = 40

Find $\overline{X}_{1.2}$

Sol :- $\overline{X}_{1.2} = \frac{\overline{X}_1 \cdot N_1 \ (+) \ \overline{X}_2 \cdot N_2}{N_1 + N_2}$

$= \frac{40 \times 60 \ (+) \ 35 \times 40}{60 \ (+) \ 40}$

$= \frac{2400 \ (+) \ 1400}{100}$

$= \frac{3800}{100} = 38$

* IMPORTANT QUESTIONS *

Q1 :- Mean of 10 items is 15. The values of 9 items are 5, 30, 20, 12, 15, 3, 10, 2, 18. Find the 10th term : x

(Individual Series)

(Individual Series)

Sol:- $\bar{X} = \frac{\Sigma X}{N} = \frac{\text{Sum of all observations}}{\text{No. of observations}}$

$$15 = \frac{5+30+20+12+15+3+10+2+18+x}{10}$$

$$150 = 115 + x$$

$$\therefore x = 35$$

Q2 -

Marks	5	15	25	35	45	55
No. of students	5	7	12	???	5	3

Calculate missing frequency when A.M = 30

Sol:-

X	f	fX
5	5	25
15	7	105
25	12	300
35	x	35x
45	5	225
55	3	165
	32 + x	820 + 35x

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

$$30 = \frac{820 + 35x}{32 + x}$$

$$960 + 30x = 820 + 35x$$

$$960 - 820 = 35x - 30x$$

$$140 = 5x$$

$$\therefore x = \frac{140}{5} = 28$$

Q3 -

C.I	0-10	10-20	20-30	30-40	40-50	50-60
f	5	??	12	18	5	3

Q3 -

C. I	0-10	10-20	20-30	30-40	40-50	50-60
f	5	??	12	18	5	3

Find Missing frequency when A.M = 29

Sol:-

C. I	f	m	f m
0 - 10	5	5	25
10 - 20	x	15	15x
20 - 30	12	25	300
30 - 40	18	35	630
40 - 50	5	45	225
50 - 60	3	55	165
	43+x		1345 + 15x

$$\bar{X} = \frac{\Sigma f m}{\Sigma f}$$

$$29 = \frac{1345 + 15x}{43 + x}$$

$$1247 + 29x = 1345 + 15x$$

$$29x - 15x = 1345 - 1247$$

$$14x = 98$$

$$x = \frac{98}{14} = 7$$

Q4 :- **Corrected Mean** :- The A.M of 40 items was calculated as 265. But while calculating it, one item 115 (सही) was misread as 150 (गलत). Find corrent A.M.

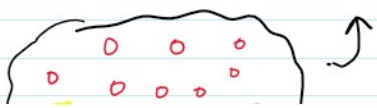
Sol:-

$$\bar{X} = \frac{\Sigma X}{N}$$

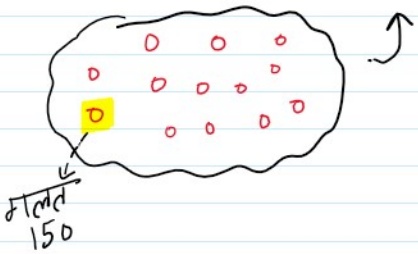
$$265 = \frac{\Sigma X}{40}$$

Incorrect ΣX $= 265 \times 40 = 10600$

Sum of all observations



Sum of all observations



$$\text{Correct } \Sigma X = 10,600 \ (-)\ 150 + 115$$

$$= 10,565$$

$$\therefore \text{ Correct A.M} = \frac{\text{Correct } \Sigma X}{N}$$

$$= \frac{10565}{40}$$

$$= 264.12$$